

Target Tracking in Wireless Sensor Networks Using Particle Filter with Quantized Innovations

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Abstract – *Due to the bandwidth constraint of wireless sensor networks, there can be physical limitations in the communication links from sensors back to fusion center, or between sensors. In such cases, local data quantization/compression is not only a necessity, but also an integral part of the design of the sensor networks. In this paper, a target tracking approach using particle filter with quantized innovations in wireless sensor networks is proposed. The posterior Cramer-Rao lower bound for quantized innovation information received by fusion center is also given. The simulation results show the good performance of our proposed tracking approach. With a moderate small number of particles sampled at each step, we found that the tracking performance of particle filter is much better than the EKF, especially when the emitted power of each sensor is small.*

Keywords: Target tracking, particle filter, posterior Cramer-Rao lower bound, wireless sensor network.

1 Introduction

A wireless sensor networks (WSNs) consists of spatially distributed autonomous devices using sensors to cooperatively monitor physical or environmental conditions, such as temperature, sound, vibration, pressure, motion or pollutants, at different locations. Recently, with the advances in micro-electronics and wireless communications, large scale of wireless sensor networks are developed to accomplish specific tasks including data collection, environment monitoring, vehicle tracking etc. These sensor networks consist of massively distributed low-cost, low-power and small size sensor nodes, which have sensing, data precessing and communication capabilities [1].

In most existing WSNs target tracking systems, a significant number of low power sensor nodes are required to detect and to track the target. In many cases, a centralized architecture is employed in which the sensor nodes will send the information they have collected to

a more powerful fusion center which will in turn compute the estimated target trajectory. In WSNs, power consumption of sensor nodes can be divided into three domains which are sensing, communication and data processing [1]. A sensor node expends most energy in data communication. To minimize the communication cost, only limited information can be transmitted through networks, and harsh quantization is usually needed. Hence, local data processing is important in minimizing power consumption in WSNs.

Many researchers are currently engaged in developing energy-efficient algorithms for network coverage [2], decentralized detection [3] and estimation [4] by utilizing the quantized messages from sensors. To minimize the communication cost, only limited information can be transmitted through networks, and harsh quantization is usually needed. In this situation, better and more efficient quantization schemes than the uniform quantization are to be sought and the effects of the quantization error are to be evaluated. A larger quantization noise will be generated if the observed values are large, which results in larger information loss and leads to a lower estimation accuracy. An interesting distributed estimation approach based on the sign of innovation (SOI) has been developed for dynamic stochastic systems in [5] where only transmission of innovation of a single bit is required. A general multiple-level quantized innovation Kalman filter for estimation of linear dynamic stochastic systems has been presented in [6]. The solution to the optimal filter is given in terms of a simple Reccati recursion as in the standard Kalman filter.

Although Kalman filters are extremely useful in many application fields, it could show surprisingly bad performance for some practical applications since the nonlinearities in either system dynamic model or measurements model. Several nonlinear filters, including particle filter, unscented Kalman filter, batch filter and exact recursive filer, etc., can provide estimation accuracy that is vastly superior to EKF under different

scenarios [7]. A novel framework for target tracking in a WSN using particle filters has been proposed in [8]. This approach uses quantized sensor data and resource constrained WSN with non-ideal wireless channels. A Kalman like particle filter using quantized innovations has been proposed in [9]. Simulation results in this work show that the optimal performance can be achieved with moderate small number of particles. However, only linear system model has been considered in this work. In this paper, we propose a quantized innovation particle filter (QI-PF) for tracking in WSNs. The corresponding posterior Cramer-Rao lower bound (PCRLB) for QI-PF is also given. The simulation results show the good performance of our proposed approach.

The rest of this paper is organized as follows. In Section 2, the target dynamic model is introduced and particle filter is reviewed. The quantized innovation particle filter is proposed in Section 3 and posterior Cramer-Rao lower bound (PCRLB) is given in Section 4. Several simulations are presented in Section 5 to show the performance of our method. Concluding remarks are made in Section 6.

2 Problem Formulation

2.1 System Model

We consider a standard process model for single target moving in a two-dimensional Cartesian coordinate plane. Target dynamics is defined by 4-dimensional state vector

$$x_k = [\xi_k \ \dot{\xi}_k \ \eta_k \ \dot{\eta}_k]'$$

where ξ_k and η_k denote the coordinates of the target in the horizontal and vertical directions; $\dot{\xi}_k$ and $\dot{\eta}_k$ denote the corresponding velocity of two directions respectively at time k . The superscript $'$ denotes the transpose operation. The model for target movement is described by

$$x_k = Fx_{k-1} + w_k, \quad (1)$$

where

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and w_k is the process noise which is assumed to be white, zero-mean and Gaussian with the following covariance matrix

$$Q = q \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^3}{3} & \frac{T^2}{2} \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix},$$

where T denotes the sampling interval and q the process noise parameter.

The measurement model, which is called attenuation model, for the i -th sensor can be described as

$$\begin{aligned} y_k^i &= h_i(x_k) + v_k^i \\ &= \frac{\Psi d_0^\alpha}{d_{ik}^\alpha} + v_k^i, \quad i = 1, 2, \dots, N \end{aligned} \quad (2)$$

where $h_i(\cdot)$ is a function that models the received signal power by the i -th sensor and v_k^i is a Gaussian white noise process independent from w_k and independent from noise samples of other sensors, $v_k^i \sim N(0, \sigma^2)$; Ψ is the emitted power of the target measured at a reference distance d_0 ; α is an attenuation parameter that depends on the transmission medium; d_{ik} is the distance between the target and the i -th sensor:

$$d_{ik} = \sqrt{(\xi_k - \xi^i)^2 + (\eta_k - \eta^i)^2}, \quad (3)$$

where (ξ^i, η^i) and (ξ_k, η_k) are the coordinates of the i -th sensor and the target at time k respectively. These target dynamic model and measurement model are widely used for target tracking in WSNs [8].

The general assumptions we make about our measurement model are as follows. Without loss of generality, the reference distance d_0 is set to be 1, and the attenuation parameter α is assumed to be 2 respectively. The fusion center is assumed to have all the information about the noise statistics. Note that the attenuation parameter and the sensor noise statistics can be empirically determined off-line. We also assume that sensor noises as well as the wireless links between the sensors and the fusion center are independent across sensors.

2.2 Particle Filter

Particle filter, also known as sequential Monte Carlo methods (SMC), are sophisticated estimation techniques based on simulation. With sufficient samples, particle filter approach the Bayesian optimal estimate, so they can be made more accurate than either the EKF or UKF. The key idea of particle filter is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights. As the number of samples becomes very large, this MC characterization becomes an equivalent representation to the usual functional description of the posterior pdf, and the SIS filter approaches the optimal Bayesian estimate. We summarize the Particle Filter as follows:

- 1) Sample the initial particle $\tilde{x}_0^{(i)} \sim p(x_0)$,

Assign initial importance weights $\tilde{\omega}_0^{(i)} = \frac{1}{M}$, M is the total number of particles.

- 2) At time k , select M particles indices $j_i \in \{1, \dots, M\}$ according to weights:

$$\{w_{k-1}^{(j)}\}_{1 \leq j \leq M}.$$

Set $x_{k-1}^{(i)} = \tilde{x}_{k-1}^{(j_i)}$, and $\omega_{k-1}^{(i)} = 1/M$, $i = 1, \dots, M$.

- 3) At time k , propagate

$$\tilde{x}_k^{(i)} \sim q_k \left(x_k^{(i)} | \tilde{x}_{k-1}^{(i)}, y_k \right)$$

and compute weight

$$\tilde{w}_k^{(i)} = w_k^{(i)} \frac{g(y_k | \tilde{x}_k^{(i)}) f(\tilde{x}_k^{(i)} | \tilde{x}_{k-1}^{(i)})}{q_k(\tilde{x}_k^{(i)} | \tilde{x}_{k-1}^{(i)}, y_k)}.$$

- 4) Normalize weights

$$w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{j=1}^M \tilde{w}_k^{(j)}}.$$

- 5) Measurements update

$$\hat{x}_{k|k} = \sum_{i=1}^M w_k^{(i)} \tilde{x}_k^{(i)}.$$

where the distribution $f(\cdot|\cdot)$ is called transition density which is derived from the dynamic model (1) and the distribution $g(\cdot|\cdot)$ is called observation density which is derived from the measurement model (2). $q_k(\cdot|\cdot, \cdot)$ is the importance density which the particles drawn from.

The commonly used particle filter, called bootstrap filter proposed by [11], chooses the state transition density f , or ‘‘prior kernel’’ as importance distribution. The importance weight is then simplified to

$$w_k^{(i)} \propto w_{k-1}^{(i)} g(y_k | \tilde{x}_k^{(i)}).$$

A distinctive feature of the bootstrap filter is that the incremental weight does not depend on the past trajectory of the particles but only on the likelihood of the observation, $g(y_k | x_k)$. The use of the prior kernel is popular because sampling is often straightforward, and computing the incremental weight simply amounts to evaluating the conditional likelihood of the new observation given the updated particle position.

3 Quantized Innovation Particle Filter

It is well known that the Kalman filter provides an optimal solution to the Bayesian sequential problem, and no algorithm can ever do better tracking performance than Kalman filter in this linear/Gaussian environment assumption [10]. In the case of nonlinear/Gaussian systems, extended Kalman filter (EKF)

can be used to provide a suboptimal solution by linearizing the nonlinear state dynamics and/or measurement equations locally. However, it has been shown in [12] that, even for linear/Gaussian systems, when the sensor measurements are quantized, EKF fails to provide an acceptable performance especially when the number of quantization levels is small. For our tracking problem, in addition to the nonlinear mapping from the target state to the sensor observations (2) and the quantization process at the sensor nodes also results in a highly nonlinear system.

At the time step k , the i -th sensor makes an observation y_k^i . Furthermore, the i -th sensor can receive one-step prediction of the state, i.e. $\hat{x}_{k|k-1}$, from the fusion center. Based on this information, the i -th sensor can compute its own innovation

$$\epsilon_k^i := y_k^i - h_i(\hat{x}_{k|k-1}),$$

and the corresponding normalized innovation

$$\bar{\epsilon}_k^i = \frac{\epsilon_k^i}{\sigma_i}.$$

Since the error covariance is not available for each sensor, we use the covariance of measurements noise to approximate the variance of innovation for each sensor. Under the Gaussian assumption of the innovation, the normalized innovation $\bar{\epsilon}(n)$ is considered approximately follows a standard normal distribution. We shall quantize the normalized innovation into $(2L+1)$ levels, 0 and $\pm\gamma_j^i$, $j = 1, 2, \dots, L$. We consider a symmetric quantizer for $\bar{\epsilon}_k^i$ given by

$$b_k^i := \begin{cases} \lambda_L^i, & \gamma_L^i < \bar{\epsilon}_k^i \\ \lambda_{L-1}^i, & \gamma_{L-1}^i < \bar{\epsilon}_k^i \leq \gamma_L^i \\ \vdots & \vdots \\ \lambda_1^i, & \gamma_1^i < \bar{\epsilon}_k^i \leq \gamma_2^i \\ 0, & -\gamma_1^i < \bar{\epsilon}_k^i \leq \gamma_1^i \\ -\lambda_1^i, & -\gamma_2^i < \bar{\epsilon}_k^i \leq -\gamma_1^i \\ \vdots & \vdots \\ -\lambda_L^i, & \bar{\epsilon}_k^i \leq -\gamma_L^i \end{cases} \quad (4)$$

It should be noted that due to the presence of quantizer, the innovation may not remain Gaussian. However, as in [5], it is assumed that the innovation is approximately Gaussian. The optimal quantization level γ_j^i , which means the minimum distortion with quantization, can be chosen according to [13]. Denote the measurements that the fusion center receives from the sensor nodes as $b_k = \{b_k^1, b_k^2, \dots, b_k^N\}$. From a Bayesian perspective, the tracking problem is to recursively calculate some degree of belief in the state x_k at time k , taking different values, given the data $\mathbf{b}_{0:k} = \{b_1, b_2, \dots, b_k\}$ up to time k . Thus, it is required to construct the pdf $p(x_k | \mathbf{b}_{0:k})$. It is assumed that the initial pdf $p(x_0)$ of the state vector, which is also known as the prior, is

available (being the set of no measurements). Then, the pdf $p(x_k|\mathbf{b}_{0:k})$ may be obtained recursively.

For the tracking problem in wireless sensor network, we employ the bootstrap filter to solve our nonlinear bayesian sequential estimation problem. We assume the communication channel from sensor node to the fusion center is perfect and the fusion center can broadcast the predicted state to the sensor nodes. The innovations for each sensors are assumed to be Gaussian distribution with zero mean. We can use the sensor measurement model (2) with the Gaussian noise assumption and the quantization model (4) to derive the probability of a quantized sensor measurement taking a specific value conditioned on the target state, i.e., $p(b_k^i|x_k)$,

$$\begin{aligned} p\{b_k^i = \pm l|x_k, \hat{x}_{k|k-1}\} &= p\{\gamma_l^i < \pm(y_k^i - h_i(\hat{x}_{k|k-1})) \leq \gamma_{l+1}^i|x_k, \hat{x}_{k|k-1}\} \\ &= p\{\gamma_l^i < \pm(v_k^i + h_i(x_k) - h_i(\hat{x}_{k|k-1})) \\ &\quad \leq \gamma_{l+1}^i|x_k, \hat{x}_{k|k-1}\} \\ &= q\left(\frac{\gamma_l^i - (h_i(x_k) - h_i(\hat{x}_{k|k-1}))}{\sigma_i}\right) \\ &\quad - q\left(\frac{\gamma_{l+1}^i - (h_i(x_k) - h_i(\hat{x}_{k|k-1}))}{\sigma_i}\right) \end{aligned}$$

where $q(\cdot)$ is the complementary distribution function of the standard Gaussian distribution. Since sensor noises and the wireless links are assumed to be independent, the likelihood that the fusion center receives the quantized data Z_k can be written as

$$p(b_k|x_k) = \prod_{i=1}^N p(b_k^i|x_k) \quad (5)$$

For the QI-PF, the likelihood function derived in (5) is the only remaining distribution required for the SIR particle filtering algorithm to work. At a given time step k , it is straightforward from (5) to calculate the likelihood function for each particle \tilde{x}_k^j

$$p(b_k|\tilde{x}_k^{(j)}) = \prod_{i=1}^N p(b_k^i|\tilde{x}_k^{(j)}), \quad j = 1, \dots, M \quad (6)$$

4 Posterior Cramer-Rao Lower Bounds

Denote the all available measurements up to time k as $y_{0:k} = \{y_1, y_2, \dots, y_k\}$ and the estimator of the state vector x_k according to $y_{0:k}$ as $\hat{x}(y_{0:k})$. Then, the mean square error (MSE) matrix of the estimation error at time k is bounded by the PCRLB J_k^{-1} [14]

$$P_{k|k} = E\{(\hat{x}(y_{0:k}) - x_k)((\hat{x}(y_{0:k}) - x_k)')\} \geq J_k^{-1} \quad (7)$$

where J_k is the fisher information matrix (FIM) which can be calculated in a recursive,

$$J_{k+1} = D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1}D_k^{12},$$

where

$$D_k^{11} = E\{-\Delta_{x_k}^{x_k} \log p(x_{k+1}|x_k)\} \quad (8)$$

$$D_k^{12} = E\{-\Delta_{x_k}^{x_{k+1}} \log p(x_{k+1}|x_k)\} \quad (9)$$

$$D_k^{21} = E\{-\Delta_{x_{k+1}}^{x_k} \log p(x_{k+1}|x_k)\} = (D_k^{12})' \quad (10)$$

$$\begin{aligned} D_k^{22} &= E\{-\Delta_{x_{k+1}}^{x_{k+1}} \log p(x_{k+1}|x_k)\} \\ &\quad + E\{-\Delta_{x_{k+1}}^{b_{k+1}} \log p(b_{k+1}|x_{k+1})\}. \end{aligned} \quad (11)$$

The operator Δ is defined as

$$\Delta_{\Psi}^{\Theta} = \nabla_{\Psi} \nabla_{\Theta}',$$

where ∇ is the gradient operator expressed as

$$\nabla_x = \left[\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_r} \right]'$$

In the case that the quantized information are received by the fusion center, the available quantized data up to time k are denoted as $\mathbf{b}_{0:k} = \{b_1, b_2, \dots, b_k\}$ and the estimator of the state vector x_k according to $\mathbf{b}_{0:k}$ as $\hat{x}(\mathbf{b}_{0:k})$. Then, the mean square error (MSE) matrix of the estimation error at time k is bounded by the PCRLB J_k^{-1} [14]

$$P_{k|k} = E\{(\hat{x}(\mathbf{b}_{0:k}) - x_k)((\hat{x}(\mathbf{b}_{0:k}) - x_k)')\} \geq J_k^{-1}$$

where J_k is the fisher information matrix (FIM).

In our tracking problem, since the target dynamic model (1) in our tracking problem is linear, (8)-(11) can be written as

$$\begin{aligned} D_k^{11} &= F^T Q^{-1} F \\ D_k^{12} &= -F^T Q^{-1} = (D_k^{21})' \\ D_k^{22} &= Q^{-1} + \tilde{D}_k^{22} \\ &= Q^{-1} + E\{-\Delta_{x_{k+1}}^{x_{k+1}} \log p(b_{k+1}|x_{k+1})\} \end{aligned}$$

Recalling the calculation of conditional probability (5), we have

$$\begin{aligned} p(b_{k+1}|x_{k+1}) &= \prod_{i=1}^N p(b_{k+1}^i|x_{k+1}) \\ &= \prod_{i=1}^N \left[Q \left(\frac{\gamma_l^i - (h_i(x_k) - h_i(\hat{x}_{k|k-1}))}{\sigma_i} \right) \right. \\ &\quad \left. - Q \left(\frac{\gamma_{l+1}^i - (h_i(x_k) - h_i(\hat{x}_{k|k-1}))}{\sigma_i} \right) \right] \end{aligned}$$

\tilde{D}_k^{22} does not have a closed-form solution, and the Monte Carlo integration methodology should be employed [8].

5 Simulations

In this section, we evaluate the tracking performance of our QI-PF target tracking approaches developed in

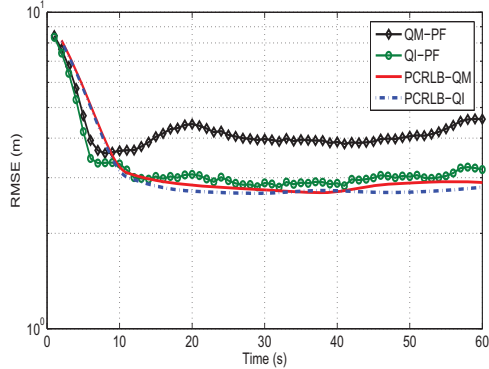


Figure 1: RMSEs of the location estimates of the target by QI-PF and QM-PF with 100 Monte Carlo trials

Sections 3. Sensors are assumed to be grid deployed in a $100 \times 100 m^2$ area and the sensors are assumed to be identical. The number of sensors is 25. For simplicity, each sensor is assumed to employ identical thresholds for quantization and the sensor measurements noise are independent with unit variance, i.e. $\sigma_i = 1$. For the target dynamic model, the following scenario is selected where all units are in meters, seconds and meters per second corresponding to distance, time and velocity measurements, respectively. The initial state distribution of the target $p(x_0)$ is assumed to be Gaussian with mean

$$\mu_{x_0} = (-75, 2, -75, 2)',$$

and covariance

$$\Sigma_{x_0} = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

The target motion follows a constant velocity model (1) with a process noise parameter $q = 0.04$. Measurements are assumed to be taken at an interval with 1 second, i.e. $T = 1$. The tracking step length is 60 s. The reference distance d_0 is set to 1 and the attenuation parameter α is also set to 1. The emitted power Ψ of each sensor is 50.

The main criterion of tracking performance is the root mean square error (RMSE) which is defined as the Euclidean distance of the true target position and the estimated position,

$$\varepsilon_k = \sqrt{(\xi_k - \hat{\xi}_k)^2 + (\eta_k - \hat{\eta}_k)^2},$$

where $(\hat{\xi}_k, \hat{\eta}_k)$ is the estimate of (ξ_k, η_k) at k -th step.

The tracking performance of quantized particle filter, which including quantized measurements PF (QM-PF) [8] and our proposed QI-PF, are shown in Figure 1. The PCRLBs for quantized innovation and measurements are different since the measurements that received at

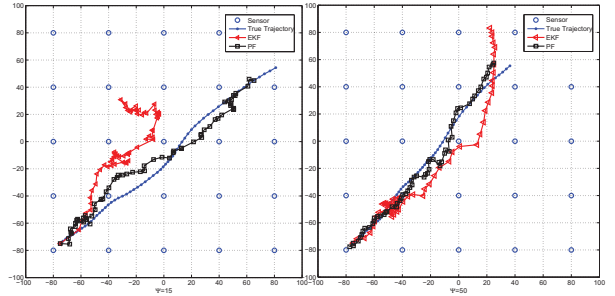


Figure 2: A realization of a target trajectory and its estimates by EKF and PF with emitted power $\Psi = 15$ and $\Psi = 50$.

fusion center are different as well as the calculations of (12). The tracking performance of QI-PF is better than QM-PF. In addition, the RMSE of the QI-PF is quite close to PCRLB for quantized innovation while the QM-PF is much farther away from the PCRLB for quantized measurements. That is because the optimal quantization level for QM-PF is difficult to obtain while the normalized innovation for QI-PF can be obtained according to [13].

The tracking trajectories of quantized innovation EKF (QI-EKF) [6] and QI-PF with different target emitted power are shown in Figure 2. The emitted power Ψ of target is set to be 15 and 50, respectively. From the tracking trajectories we can see that the EKF has significant performance degradation for the scenario when the emitted power is decreased. The RMSE of the location estimates of the target by QI-EKF and QI-PF with 100 Monte Carlo trials is shown in Figure 3. The tracking performance for QI-EKF is slightly worse than the performance of QI-PF when the emitted power of target is $\Psi = 50$. However, when the emitted power is decreased to 15 the QI-EKF will be divergent.

6 Conclusion

In this paper, we have considered the quantization filtering methods for target tracking in wireless sensor networks. Since the nonlinearity of the measurement equation for target tracking, we have proposed a QI-PF for nonlinear filtering. The PCRLBs for quantized information received by fusion center are given. The simulation results showed that the quantization of innovation is better than the quantization of measurements. Furthermore, with a considerable particles sampled of each step, we found that the tracking performance of particle filter is much better than the EKF, especially when the emitted power of each sensor is small.

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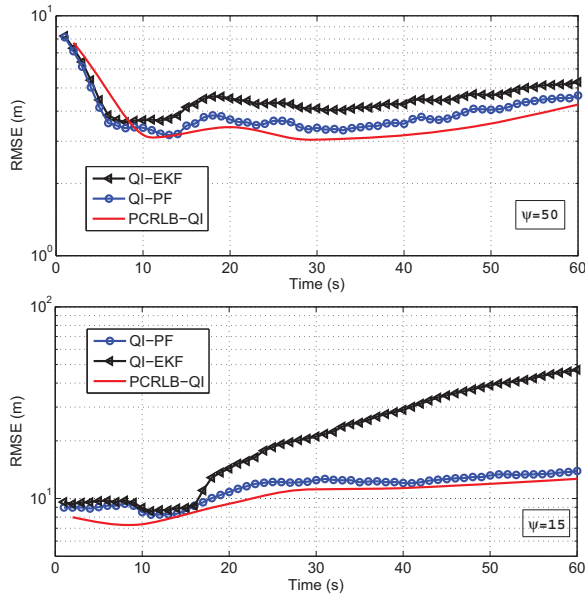


Figure 3: RMSE of the target location estimates of by QI-EKF and QI-PF with 100 Monte Carlo trials for different emitted power Ψ .

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